Engineering Notes

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A High-Accuracy Relationship between Geocentric Cartesian Coordinates and Geodetic Latitude and Altitude

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Nomenclature

a = semimajor axis of the Earth
b = semiminor axis of the Earth
ϕ = geodetic latitude of vehicle
h = geodetic altitude of vehicle
E = equatorial component of position of vehicle in geocentric Cartesian system
F = polar component of position of vehicle in geocentric Cartesian system

Introduction

THE purpose of this Note is to derive a set of equations to convert the geodetic latitude and altitude of a vehicle into a geocentric Cartesian system, and vice versa. Because of the fact that these equations are to be used on high speed digital computers, the following requirements were levied on the equations: simplicity for small storage requirements, speed due to the great number of times the equations would be used in the program, and accuracy to be kept to the order of magnitude of the least significant digit in the computer, assuming input values of the same accuracy. It is concluded that the method described in this Note offers advantages in speed, accuracy, and storage requirements over other comparable methods of computation.

Computation of Geocentric Coordinates from Geodetic Latitude and Altitude

Assume a plane through the polar axis of the Earth and containing the vehicle (V). The surface of the Earth is then represented as an ellipse with semimajor axis a and semiminor axis b (see Fig. 1). The eccentricity of this ellipse is given by

$$e^2 = 1 - (b/a)^2 \tag{1}$$

From V drop a line normal to the ellipse (at S) and continue it until it intersects the y axis (at P). The length of the line VS is the geodetic altitude (h) of the vehicle. The slope of this line is the tangent of the geodetic latitude of the vehicle and is

$$\tan\phi = a^2y_1/b^2x_1 \tag{2}$$

N is the length of that portion of the normal line connecting points S and P. The intersection of this line with the y axis at P is derived as follows. The equation of the line yields

$$y = (a^2y_1/b^2x_1)x + y_2$$

or

$$y_2 = y - (a^2y_1/b^2x_1)x$$

Upon evaluating this equation at S and reducing, the equation becomes

$$y_2 = y_1(1 - a^2/b^2) \tag{3}$$

If N equals the length of line PS, $N = [x_1^2 + (y_1 - y_2)^2]^{1/2}$ which, upon substitution of Eq. (3) and reducing, yields

$$N = [x_1^2 + y_1^2(a/b)^4]^{1/2} \tag{4}$$

Removing x_1^2 from under the radical and using Eq. (2), we have $N = x_1(1 + \tan^2\phi)^{1/2}$. Solving this for x_1 yields

$$x_1 = N\cos\phi \tag{5}$$

y_1 can be derived in terms of N and ϕ using Eq. (4) as follows:

$$N = (a/b)^2y_1(1 + [b^2x_1/a^2y_1]^2)^{1/2}$$

Substitution of Eqs. (1) and (2) yields

$$N = y_1/(1 - e^2)(1 + \cot^2\phi)^{1/2}$$

Solving for y_1 yields

$$y_1 = N(1 - e^2)\sin\phi \tag{6}$$

The general equation of the ellipse evaluated at S is

$$x_1^2/a^2 + y_1^2/b^2 = 1$$

Solving for a gives

$$a = x_1(1 + a^2y_1^2/b^2x_1^2)^{1/2}$$

Substituting for x_1 and y_1 using Eqs. (2) and (5) yields

$$a = N\cos\phi(1 + [b/a]^2\tan^2\phi)^{1/2}$$

or, rearranging,

$$a = N\{1 - [1 - (b/a)^2]\sin^2\phi\}^{1/2}$$

and finally, using Eq. (1), $a = N(1 - e^2\sin^2\phi)^{1/2}$. Solving for N, this yields

$$N = a/(1 - e^2\sin^2\phi)^{1/2} \tag{7}$$

The equatorial component of altitude is

$$h_E = h\cos\phi \tag{8}$$

and the polar component is

$$h_F = h\sin\phi \tag{9}$$

Summing Eqs. (5) and (8) and Eqs. (6) and (9) yields the desired equations. The entire set of equations needed for this transformation is as follows:

$$N = a/(1 - e^2\sin^2\phi)^{1/2} \tag{10a}$$

$$E = (N + h)\cos\phi \tag{10b}$$

$$F = [N(1 - e^2) + h]\sin\phi \tag{10c}$$

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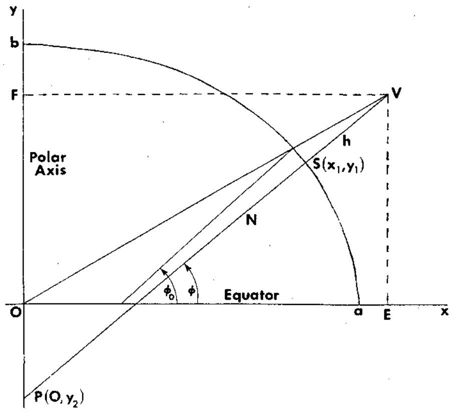

Fig. 1 Illustration of conversion geometry.

Computation of Geodetic Latitude and Altitude from Geocentric Coordinates

Assume a plane through the polar axis of the Earth and containing the vehicle (V) as shown in Fig. 1. From Eq. (10), E and F can be rewritten to eliminate N as follows:

$$E = [a/(1 - e^2 \sin^2\phi)^{1/2} + h] \cos\phi \tag{11}$$

$$F = [a(1 - e^2)/(1 - e^2 \sin^2\phi)^{1/2} + h] \sin\phi \tag{12}$$

h can be eliminated from these equations by dividing Eq. (11) by $\cos\phi$ and Eq. (12) by $\sin\phi$ to eliminate the brackets and differencing them as follows:

$$(E/\cos\phi) - (F/\sin\phi) = ae^2/(1 - e^2 \sin^2\phi)^{1/2}$$

Newton's method of determining roots of an equation may now be used by letting

$$f(\phi) = \frac{ae^2}{(1 - e^2 \sin^2\phi)^{1/2}} - \frac{E}{\cos\phi} + \frac{F}{\sin\phi} \tag{13a}$$

$$f'(\phi) = \frac{ae^4 \sin\phi \cos\phi}{(1 - e^2 \sin^2\phi)^{3/2}} - \frac{E \sin\phi}{\cos^2\phi} - \frac{F \cos\phi}{\sin^2\phi} \tag{13b}$$

If the first approximation of ϕ is the geodetic equivalent of the radius vector slope, i.e.,

$$\phi_0 = \tan^{-1}(a^2F/b^2E) \tag{14}$$

and the ith correction on ϕ is

$$\phi_i = \phi_{i-1} - f(\phi_{i-1})/f'(\phi_{i-1}) \tag{15}$$

then the correction process should be repeated until $f(\phi_{i-1})/f'(\phi_{i-1})$ becomes less than some predetermined allowable error. The final latitude value is then ϕ_i.

The coordinates of the point on the surface of the Earth (x_1,y_1) are determined using Eq. (10) with the newly determined ϕ and letting $h = 0$. h can now be computed as follows:

$$A = [(E - x_1)^2 + (F - y_1)^2]^{1/2} \tag{16a}$$

$$B = x_1(E - x_1) + y_1(F - y_1) \tag{16b}$$

$$h = A \text{ with the sign of } B \tag{16c}$$

This sign computation allows points to be computed that are below the surface of the Earth.

Checkout Results

Checkout of these equations was done on an IBM 360/65 using double precision (16 decimal places) throughout. Altitudes were checked from -1000 ft–2×10^{16} ft with each altitude tested having a latitude sweep from pole to pole. The method used was to input the latitude and altitude, convert them to the geocentric Cartesian system by means of Eq. (10), then convert them back to latitude and altitude by means of Eqs. (13–16).

A similar computation was done with three other methods of computing the geodetic latitude and altitude from geocentric Cartesian coordinates.[1–3] The results of these computations are shown in Table 1.

A significant point noted during the checkout process was that in the method of Purcell and Cowan the recommended equations use a binomial expansion of one of the derived equations [Eq. (29) instead of Eq. (17)]. If this approximation is used, an altitude error of 2.033 ft will occur for a latitude of 0°, and this error will not drop below 1 ft until a latitude of almost 30° has been reached, for all altitudes below 0.2×10^{16} ft.

Conclusions

The requirements of the equations as stated in the introduction were compared to the results of the checkout of these four methods. It is obvious from Table 1 that the methods of Berger and Ricupito and of Purcell and Cowan, though accurate enough for desk calculator work, do not meet the desired accuracy. The remainder of this Note, therefore, shall confine itself to the method of Heffron and Watson and to that method described by the author.

The method of Heffron and Watson requires a basic 1.93 msec plus 0.90 msec for each iteration required. The number of iterations was dependent upon altitude, needing two iterations (3.73 msec) for under 100,000 ft, three iterations (4.63 msec) between 100,000 ft and 2×10^{11} ft, and one iteration (2.83 msec) for over 2×10^{11} ft altitude. The method described in this Note used 1.15 msec plus 0.70 msec for each iteration of the correction process of Eq. (15). The number of iterations also changed with altitude, requiring two iterations (2.55 msec) for under 100,000 ft, three iterations (3.25 msec) between 100,000 ft and 2×10^8 ft, and four iterations (3.95 msec) thereafter. The criterion for iteration cutoff was 0.5×10^{-14} rad for the method of Heffron and Watson, and 0.5×10^{-11} rad for the method described in this Note.

Table 1 Maximum latitude and altitude errors as functions of altitude

	Berger & Ricupito[1]		Purcell & Cowan[2]		Heffron & Watson[3]		Hedman	
Altitude, ft	$\Delta\phi$, rad	Δh, ft	$\Delta\phi$, rad	Δh, ft	$\Delta\phi$, rad	Δh, ft	$\Delta\phi$, rad	Δh, ft
-1000	0.6×10^{-7}	0.4×10^{-2}	0.1×10^{-7}	0.5×10^{-7}	0.2×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.7×10^{-8}
-10	0.6×10^{-7}	0.4×10^{-2}	0.1×10^{-7}	0.9×10^{-8}	0.4×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.6×10^{-8}
0	0.6×10^{-7}	0.4×10^{-2}	0.1×10^{-7}	0.7×10^{-8}	0.4×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.5×10^{-8}
0.5	0.6×10^{-7}	0.4×10^{-2}	0.1×10^{-7}	0.1×10^{-7}	0.4×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.4×10^{-8}
100	0.6×10^{-7}	0.4×10^{-2}	0.1×10^{-7}	0.1×10^{-7}	0.4×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.7×10^{-8}
0.1×10^6	0.6×10^{-7}	0.4×10^{-2}	0.6×10^{-7}	0.1×10^{-4}	0.4×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.6×10^{-8}
0.2×10^7	0.4×10^{-7}	0.3×10^{-2}	0.9×10^{-6}	0.2×10^{-2}	0.2×10^{-15}	0.1×10^{-7}	0.2×10^{-15}	0.7×10^{-8}
0.2×10^9	0.3×10^{-8}	0.2×10^{-2}	0.1×10^{-5}	0.25	0.9×10^{-14}	0.7×10^{-8}	0.2×10^{-15}	0.1×10^{-7}
0.2×10^{11}	0.3×10^{-10}	0.2×10^{-2}	0.1×10^{-7}	0.31	0.8×10^{-12}	0.6×10^{-5}	0.2×10^{-15}	0.6×10^{-5}
0.2×10^{13}	0.3×10^{-12}	0.2×10^{-2}	0.1×10^{-9}	0.31	0.2×10^{-12}	0.7×10^{-3}	0.3×10^{-15}	0.1×10^{-2}
0.2×10^{15}	0.4×10^{-14}	0.2×10^{-1}	0.2×10^{-11}	0.32	0.2×10^{-14}	0.8×10^{-2}	0.7×10^{-16}	0.2×10^{-1}
0.2×10^{17}	0.4×10^{-15}	5	0.1×10^{-13}	6	0.7×10^{-15}	2	0.8×10^{-16}	7

With regards to storage and accuracy, the method described in this Note used 1194 bytes of storage on the 360/65, whereas the method of Heffron and Watson required 1324 bytes, or over 10% more storage. In addition Table 1 shows that, except for the altitude range of from 0.2×10^9 ft to 0.2×10^{16} ft, the latitude error is virtually identical and, for all practical purposes, the altitude error is the same throughout. Both methods, therefore, have an over-all accuracy of 15 out of 16 digits except where noted.

References

[1] Berger, W. J. and Ricupito, J. R., "Geodetic Latitude and Altitude of a Satellite," *ARS Journal*, Vol. 30, No. 9, Sept. 1960, pp. 901–902.

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[3] Heffron, W. G., Jr. and Watson, S. B., "Relationships between Geographic and Inertial Coordinates," *Journal of Spacecraft and Rockets*, Vol. 4, No. 4, April 1967, pp. 531–532.

Cryogenic Know-How as Applied to Inground LNG Storage

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THE concept of inground storage developed from the need to find an economical means of storing large volumes (i.e., more than 400,000 barrels) of liquefied natural gas (LNG). The costs of storing such volumes in conventional double-walled, insulated metal tanks are high. Moreover, the large-scale conventional storage tanks have not yet been designed and built; consequently, there is no assurance that LNG can be stored this way in an economic manner.

Using the ground itself as a container for LNG, or any cryogen, is, in theory at least, a simple means of overcoming the problems encountered with conventional tank storage. One has merely to excavate a large, cylindrical hole in the ground, cover it with an insulated roof, and fill the hole with LNG. The earth walls surrounding the hole cool, freezing any water present in the formation, and behave, ideally, as an infinite-slab heat sink. Although heat transfer is very high initially, it eventually decreases and boil-off reaches an acceptable level.

While the technique of inground storage is theoretically simple, the reduction to practice is not. A typical inground storage system—the CAMEL plant in Arzew, Algeria—has reported boiloff losses of about 0.3% per day, which is considered acceptable for this "base load" (shipment depot type) plant. However, two similar systems in the United States, one at Hackensack, N.J., and one at Hopkinton, Mass., have been abandoned because of high boil-off losses. Another facility built for the British Gas Council at Canvey Island, Great Britain, has recently gone into full operation and LNG is being fed into the inground tanks. A stabilized boiloff value has not been determined, but indications are that boiloff will be considerably higher than had been predicted.

This Note describes some of the factors that affect the boiloff rate of inground storage systems.

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Prefreeze and Excavation

To excavate successfully a large, deep hole in the earth, one must make the containing walls strong enough to eliminate cave-ins or slides. In addition, since typical inground tanks are 100–200 ft deep, some provision must be made to control water flow into the excavation.

Saturated frozen earth is both a water barrier and a rather strong material; thus, one way to excavate an inground storage tank is to insert a number of freeze pipes just outside the perimeter of the planned excavation (see Fig. 1). Refrigerant is pumped through these pipes at a temperature of -25 to $-100°F$, and the prefreeze system must operate continuously during the entire excavation period. After an annular wall of ice and frozen earth is formed, the hole is excavated. Once LNG filling starts, the prefreezing operation can be discontinued.

Since excavation often utilizes controlled blasting, the prefreeze pipes must be constructed of a metal which is not susceptible to brittle fracture at prefreeze temperatures. The mechanical properties of the frozen soil at reduced temperatures are extremely important also, since the tank roof usually is supported by the frozen soil. Creep rates and fracture characteristics of the soil must be known, and allowances must be made for gradual creep of the loaded ice-soil medium.

Thermal Performance of Inground Tanks

The most important heat leak to the tank contents occurs from the surrounding earth through the side walls and bottom. However, radiation from the roof can also be significant unless it is insulated well enough to isolate the tank from fluctuations in ambient conditions.

Mathematical models have been used to predict the performance of existing and proposed inground storage units. Hashemi and Sliepcevich,[1] for example, have developed a computer model which can be used to estimate cooldown and long-term boiloff performance for such tanks, provided the thermal properties of the system are known and no extraneous heat sources are present.

Thermal Properties

In general, soils and rocks are anisotropic; consequently, there is considerable variation in heat capacity, thermal conductivity, and density even for adjacent samples. In order to reasonably explain these variations in the properties of interest, we calculated how quartz content and orientation with respect to direction of heat flow affect thermal conductivity in a base rock.

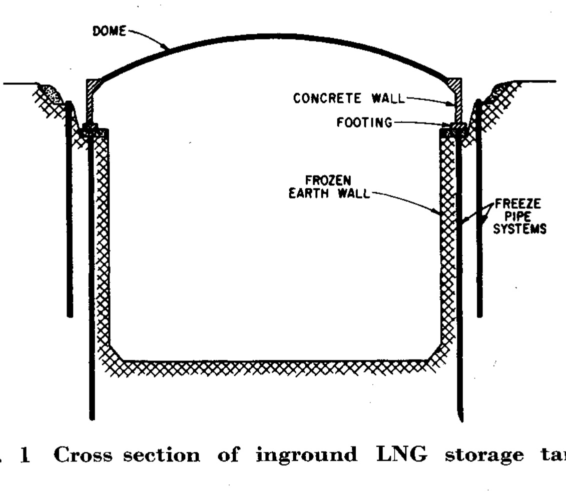

Fig. 1 Cross section of inground LNG storage tank.